# DiSSECT: Distinguisher of Standard & Simulated Elliptic Curves via Traits

Vladimir Sedlacek[1,2] Vojtech Suchanek[1] Antonin Dufka[1]
Marek Sys [1] Vashek Matyas[1]

[1]CRoCS, Masaryk University, Brno, Czech Republic

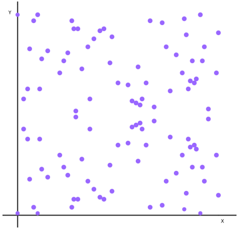
[2]Université de Picardie Jules Verne, Amiens, France

Africacrypt 2022, July 20

# Elliptic curve cryptography (ECC)

$$y^2 = x^3 + ax + b \quad \text{in } \mathbb{F}_p$$

$$k \cdot P := \underbrace{P + \cdots + P}_{k}$$



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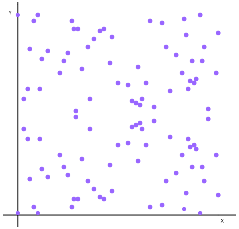

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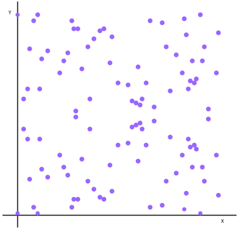

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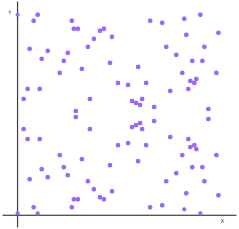

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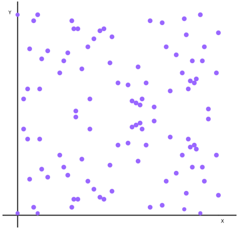

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  - How to measure their real security?

# ECDLP attacks

- Known attacks
- Unknown attacks
- Backdoors

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- `safecurves.cr.yp.to`

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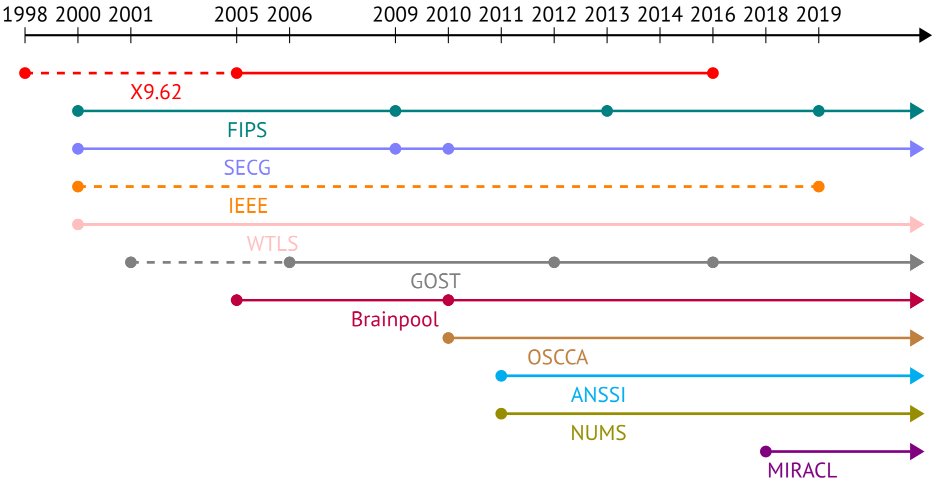
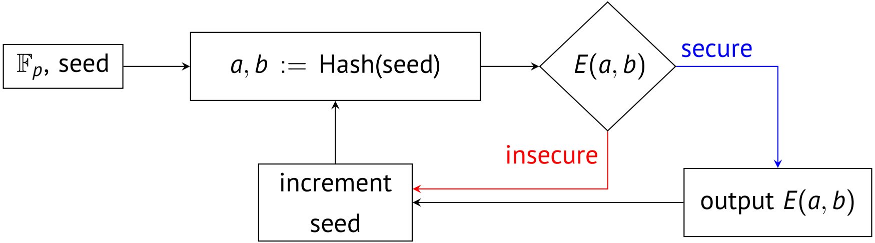
# Timeline of standard curves



# Curve generation



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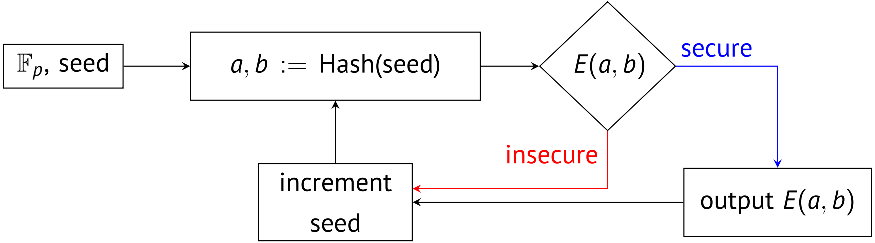

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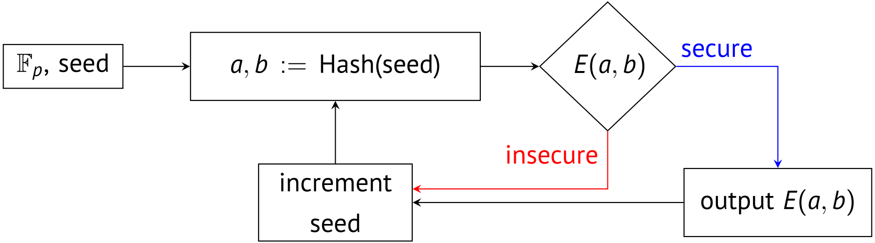

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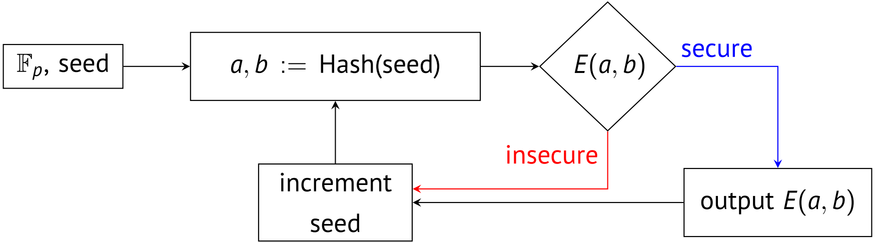

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- unknown/ambiguous origin: ANSSI FRP256v1, OSCCA SM2, GOST R

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- Transparency is the key

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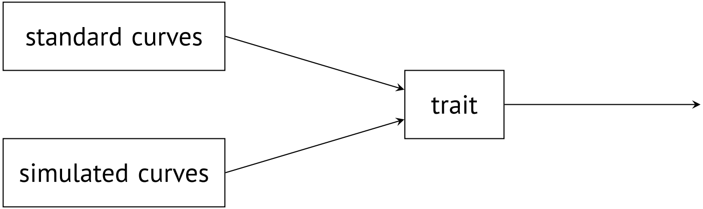
| | 256 bits | 224 bits | 192 bits | 160 bits | 128 bits |
|---|---|---|---|---|---|
| **X9.62**$_{\text{sim}}$ | 18 500 | 22 200 | 18 800 | 27 800 | 36 100 |
| **Brainpool**$_{\text{sim}}$ | 1 700 | 2 400 | 2 700 | 3 200 | 0 |
| **NUMS**$_{\text{sim}}$ | 100 | 100 | 200 | 300 | 0 |
| **Curve25519**$_{\text{sim}}$ | 100 | 0 | 400 | 300 | 0 |
| **Random** | 18 700 | 21 200 | 24 800 | 29 600 | 37 300 |

Simulation counts (>260k curves)

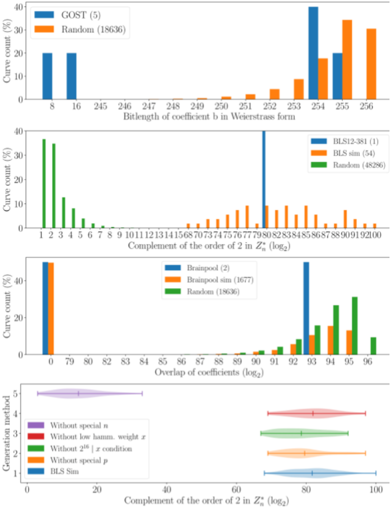
# DiSSECT: high level



standard curves

simulated curves

trait



# DiSSECT: traits

degrees
cm-field coefficients kn_factorization
degree graph twist nontrivial
full isogeny subgroup cofactor weight
containing ratio field factorization
coordinate discriminant order embedding
conductor
complement quadratic small torsion extensions twist_order
prime curve extension trace maximal torsion_extension
number cardinality least cm frobenius
relative polynomial
lower volcano prime-subgroup bitlength
conductors weierstrass upper basefield
class_number

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  - Brainpool curves often have hex coefficients overlap
- Comparison across standards often very valuable

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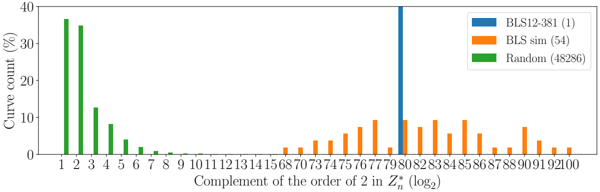

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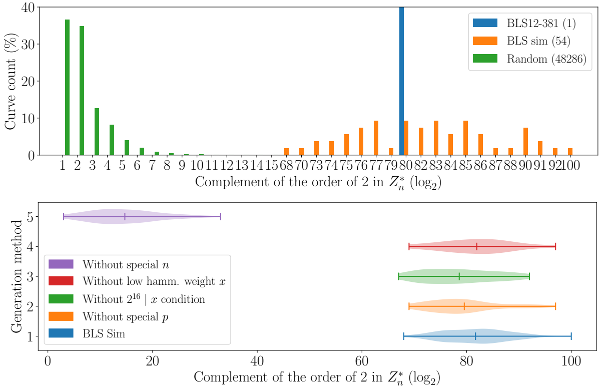

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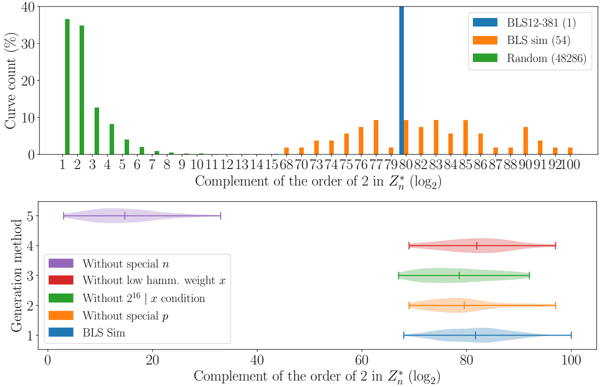

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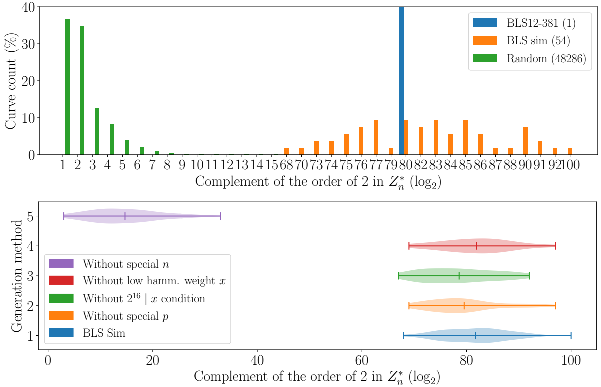
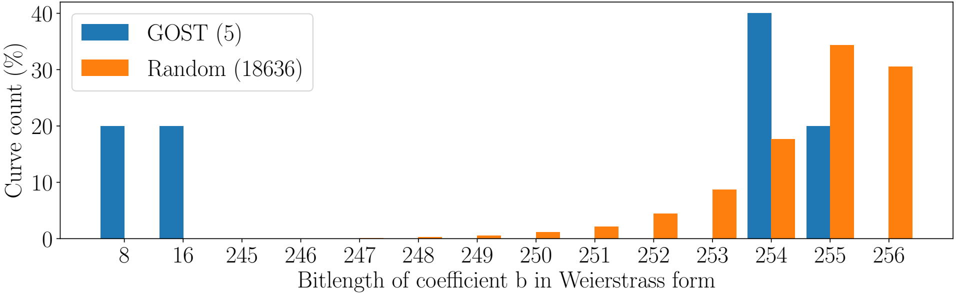


- Finding: $ord_n(2)$ is small
- Root cause: $\varphi(n) = x^2(x+1)(x-1)$ has no large factor

# DiSSECT: GOST findings



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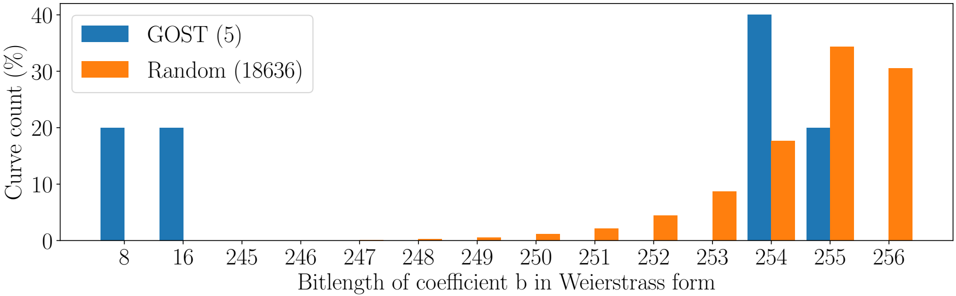

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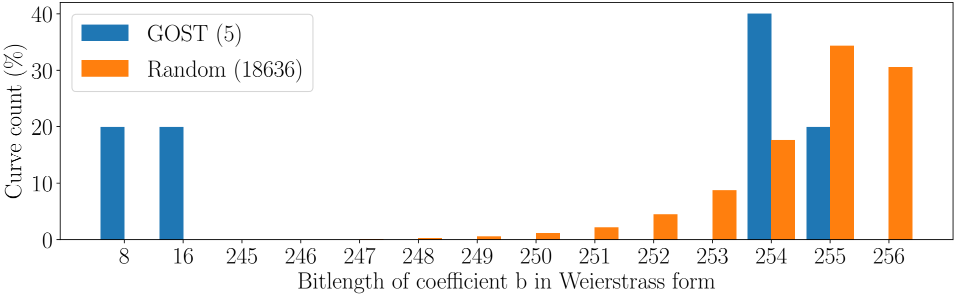

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- Conclusion: these were generated in a special way

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- WIP: pairing-friendly curves, clustering, entropy measurements,...

# Something ends, something begins

Questions and collaboration welcome!

Check out our tool and results at: https://dissect.crocs.fi.muni.cz/

CROCS

Centre for Research on
Cryptography and Security